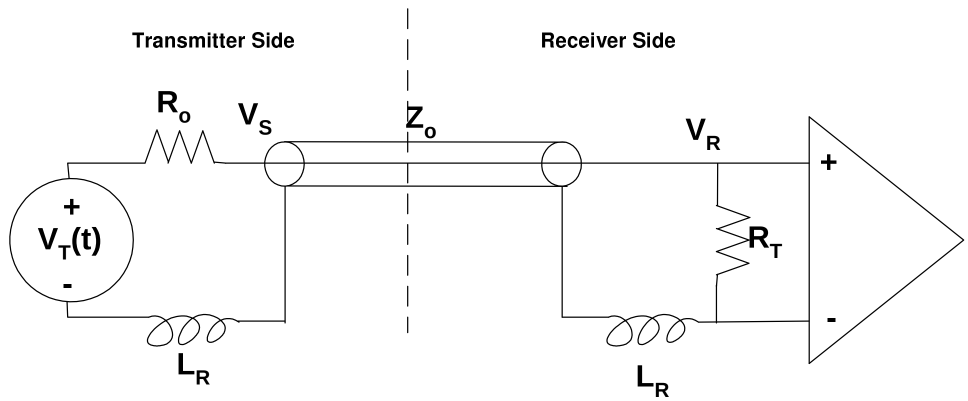
Homework 4 - Solutions

1 Problem 7-1 (Dally and Poulton)

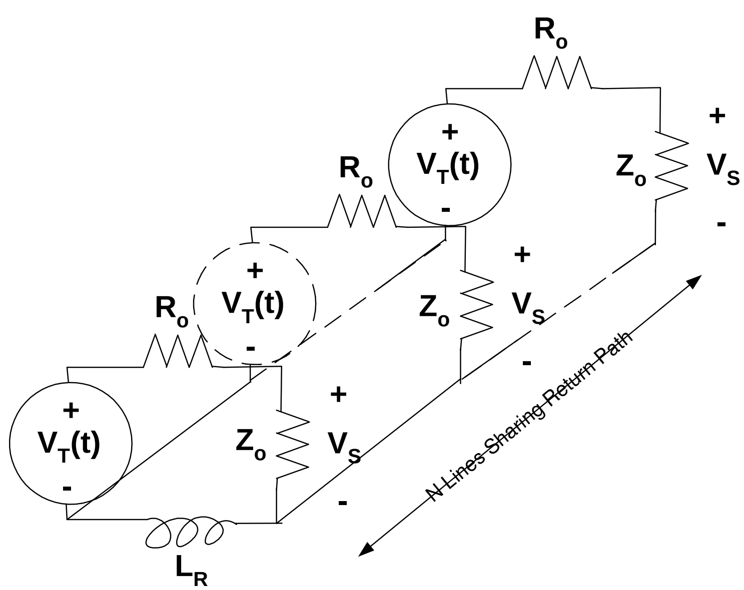
Noise Rejection of Underterminated Signaling:

Consider the underterminated signaling system shown below. As a function of return inductance, L_R, signal rise time, t_r, and parameters shown in the figure, derive an expression for the signal-return cross talk coefficient, k_{xr}. (Hint: Derive separate expressions for the return cross talk at each end of the line.)

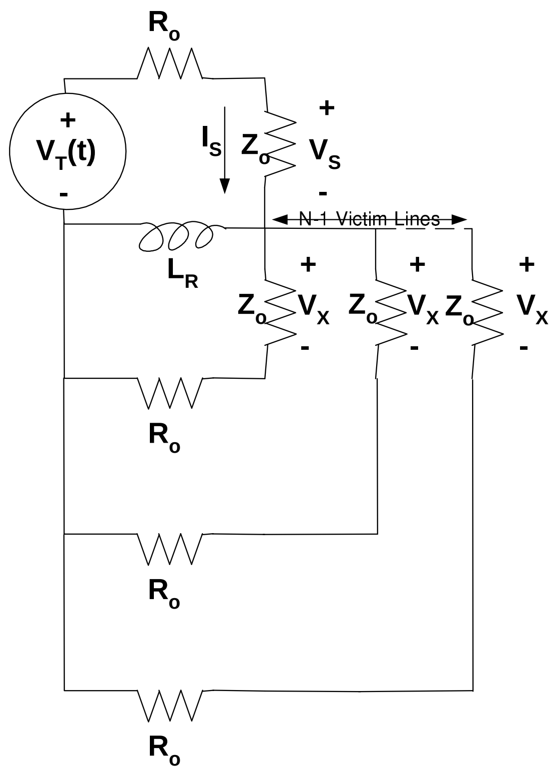


We have N lines sharing the return path. We want to solve for the signal return cross-talk, k_{xr}. As hinted in the problem, we can treat the transmitter and receiver side separately, as denoted in the figure above.

We start with the transmitter side. We begin by modeling our sytem, seen from the transmitter side, as follows:



We solve for the transmitter signal return cross-talk, K_{XRT} exactly as done in the problem session: Solving by superposition for the voltage, V_X induced across each of the other N-1 lines, we have the following circuit:



Z_X, the equivalent impedance for the parallel network, i.e. L_R in parallel with the other N-1 lines, is:

$$Z_X = Z_{RT} // \frac{R_o + Z_o}{N-1} = \frac{Z_{RT}(R_o + Z_o)}{(N-1)Z_{RT} + R_o + Z_o}$$

where Z_{RT} is the signal return impedance $\frac{L_R}{t_r}$.

The current passing through Z_o of the aggressor (and R_o) and the entire parallel network is:

$$I_S = \frac{V_S}{Z_o}$$

where V_S is the signal on the aggressor line.
Thus, the current passing through each of the N-1 victim lines is:

$$I_X = I_S \times \frac{Z_X}{Z_o + R_o} = I_S \times \frac{Z_{RT}}{(N-1)Z_{RT} + R_o + Z_o}$$

The voltage induced across each of the victim lines, V_X, is:

$$V_X = I_X \times Z_o = V_S \times \frac{Z_{RT}}{(N-1)Z_{RT} + R_o + Z_o}$$

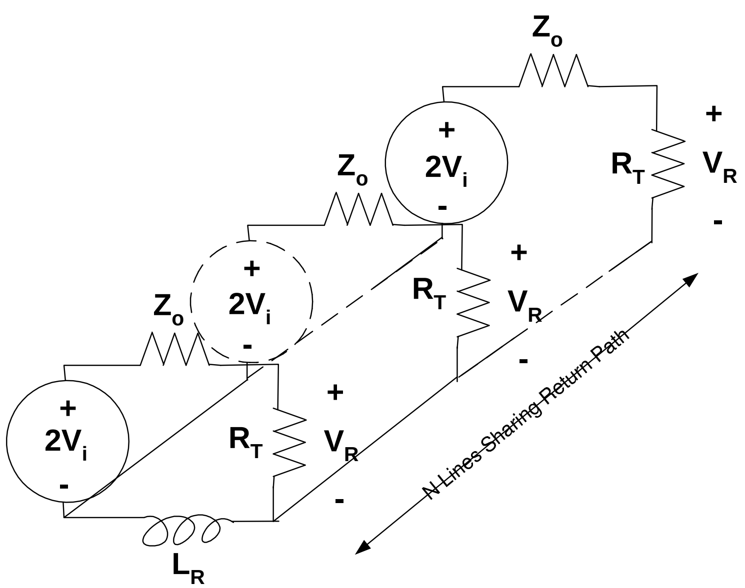
Using superposition and solving for the total voltage induced across our lines for signal return cross-talk, we get:

$$V_{X(total)} = (N-1)V_X$$

So, our signal return cross-talk for the transmitter, K_{XRT}, is:

$$\boxed{K_{XRT} = \frac{V_{X(total)}}{V_S} = \frac{(N-1)Z_{RT}}{(N-1)Z_{RT} + R_o + Z_o}}$$

Next, we solve for the receiver side. Again, we set up the equivalent circuit:

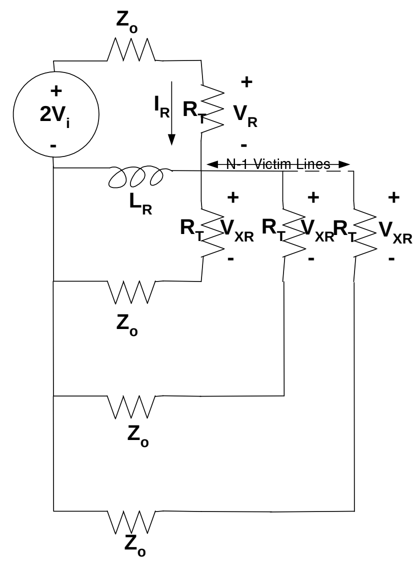


The incident voltage, V_i, however is not V_S but it is the noisy V_S, i.e. V_S with the transmitter signal return cross-talk taken into account: $V_i = V_S(1 - K_{XRT})$. Again, we solve this circuit by superposition. (Note: our termination at the receiver is matched to the line, $R_T = Z_o$.)

We solve for V_{XR}, the voltage induced across each termination due to signal return cross-talk at the receiver. $V_{XR(total)}$ is the total voltage across the termination due to the N-1 lines. I.e. $V_{XR(total)} = (N-1)V_{XR}$.

Following our previous analysis we get:

$$\boxed{K_{XRR} = \frac{V_{XR(total)}}{V_R} = \frac{(N-1)Z_{RT}}{(N-1)Z_{RT} + 2Z_o}}$$



We can now solve for $V_{XR(total)}$ by first solving for V_R. We find V_R by simple voltage division:

$$V_R = \frac{R_T}{R_T + Zo + Z_{XR}} \times 2V_i$$

,where Z_{XR} is the equivalent impedance of the parallel network in the reciever circuit. I.e.

$$Z_{XR} = \frac{2Z_{RT}Z_o}{(N-1)Z_{RT} + 2Z_o}$$

Substituting for our incident voltage and substituting Z_o for R_T we get:

$$V_R = \frac{Z_o}{2Zo + Z_{XR}} \times 2Vs(1 - K_{XRT})$$

We now solve for the total noise induced across our termination resistor:

$$V_{XR(total)} = K_{XRR} \times V_R = K_{XRR} \times \left(\frac{Z_o}{2Zo + Z_{XR}} \times 2Vs(1 - K_{XRT}) \right)$$

This total noise induced across the termination is the signal-return cross talk noise induced at the receiver. We can then solve for the overall signal-return cross talk coefficient, K_{xr}, due to N lines sharing the signal return:

$$\boxed{K_{xr} = \frac{V_{XR(total)} + V_{transmittedNoise}}{V_S}}$$

, where $V_{transmittedNoise}$ is the noise from the transmitter side, i.e. how much the voltage across V_R is reduced because of the signal-return cross talk at the receiver:

$$V_{transmittedNoise} = 2V_S \times K_{XRT} \left(\frac{Z_o}{2Z_o + Z_{XR}} \right)$$

Thus,

$$\boxed{K_{xr} = K_{XRR} \times \left(\frac{Z_o}{2Zo + Z_{XR}} \times 2(1 - K_{XRT})\right) + K_{XRT}\frac{2Z_o}{2Z_o + Z_{XR}}}$$

Plugging in the values for K_{XRT}, K_{XRR}, Z_{XR}, and Z_{RT} that we solved for previously, we obtain the overall signal-return cross talk coefficient:

$$\boxed{K_{xr} = \left(\frac{(N-1)\frac{L_R}{t_r}}{(N-1)\frac{L_R}{t_r} + 2Z_o}\right)\left(\frac{Z_o}{2Z_o + \frac{2\frac{L_R}{t_r}Z_o}{(N-1)Z_{RT}+2Z_o}}\right)\left(2\left[1 - \frac{\frac{L_R}{t_r}(N-1)}{\frac{L_R}{t_r}(N-1) + R_o + Z_o}\right]\right) + K_{XRT}\left(\frac{2Z_o}{2Z_o + \frac{2\frac{L_R}{t_r}Z_o}{(N-1)Z_{RT}+2Z_o}}\right)}$$

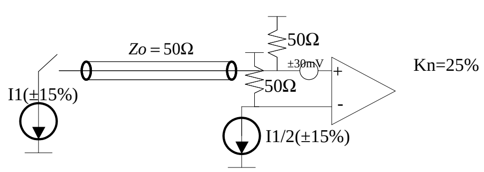
Note: if Z_{XR} is much less than Z_o we have:

$$\boxed{K_{xr} = K_{XRR}(1 - K_{XRT}) + K_{XRT}}$$

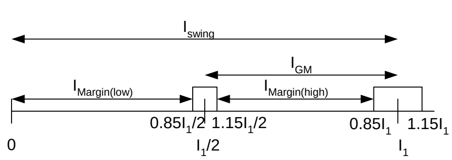
2 Problem 7-3 (Dally and Poulton)

Current References:

Consider a unipolar current-mode signaling system that encodes a 0 as $I_0 = 0$, and a 1 as I_1. The transmitter and receiver each generate their own current reference, and these references are each $\pm 15\%$ of nominal. Suppose the fixed noise sources total 30mV (600μ A assuming a 50-ohm load), and cross talk, ISI, and signal return noise give proportional noise that totals 25%. What is the effect of this variation in current reference? What is the minimum signal swing (I_1) that will satisfy a worst-case noise analysis? What is the minimum signal swing that would satisfy a worst-case noise analysis if bipolar current-mode signaling were used ($I_o = -I_1$)?



Taking into account only the transmitter and reference offsets for this unipolar signaling system, the noise breakdown is as follows:



Since our termination resistors are equal, $R_T = 50\Omega$, we can do our analysis completely in terms of current. You could also do an equivalent analysis in terms of voltage.

If we choose our reference to be $\frac{I_1}{2}$, our margin before taking into account the additional fixed and proportional noise will be:

$$I_{Margin(high)} = I_{GM} - 0.15I_1 - 0.15\frac{I_1}{2} = 0.275I_1$$

$$I_{Margin(low)} = 0.85\frac{I_1}{2} = 0.425I_1$$

Note that by choosing our reference to be $\frac{I_1}{2}$, our high and low noise margins are not equal. In our noise analysis we are limited by the smallest margin, $I_{Margin(high)}$.

The gross noise margin, I_{GM}, must be large enough to compensate for all of our noise. Equivalently, he margin calculated above, $I_{Margin(high)}$, must be large enough to compensate for the proportional noise and the fixed noise, I_N:

$$I_N = K_N I_s wing + I_{ni} = 0.25I_1 + 600\mu A$$

Thus, we have the relationship:

$$I_{GM} = \frac{I_1}{2} \geq 0.15I_1 + 0.15\frac{I_1}{2} + I_N$$

Equivalently,

$$0.275I_1 \geq 0.25I_1 + 600\mu A$$

Thus, the minimum signal swing that will satisfy a worst-case noise analysis:

$$\boxed{I_1 \geq 24mA}$$

The previous solution assumed that the proportional voltage swing on the line was just I_1. Actually, the worst case would be if the agressor had $1.15I_1$ current for its voltage swing, while this particular line had the worst case voltage swing($0.85I_1$). In this case,

$$0.275I_1 \geq 0.25(1.15I_1) + 600\mu A$$

Thus, we have:

$$-0.0125I_1 \geq 600\mu A$$

This condition is not satisfiable and will not pass a worst-case noise analysis. However, if we make our high and low margins equal by adjusting our reference signal, we get:

$$I_{ref} = \frac{0.85I_1}{2} = 0.425I_1$$

With this reference, our low and high margins are equal:

$$I_{Margin(high)} = I_1 - 0.425I_1 - 0.15I_1 - 0.15(0.425I_1) = 0.361I_1$$

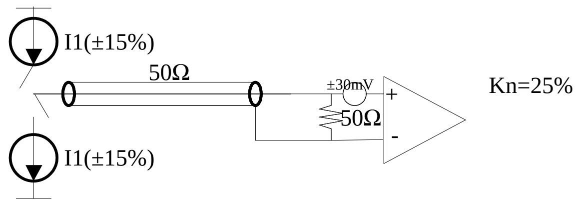
$$I_{Margin(low)} = 0.425I_1 - 0.15(0.425I_1) = 0.361I_1$$

In this case, we have:

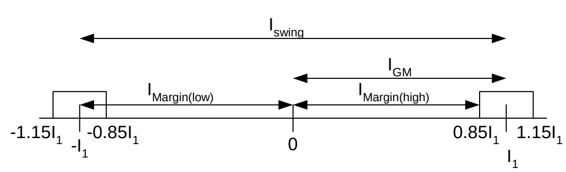
$$0.361I_1 \geq 0.25(1.15I_1) + 600\mu A$$

$$\boxed{I_1 \geq 8.14mA}$$

If we use bipolar signaling, as shown below, we remove the noise caused by the reference, since our reference is now 0 mA.



Again, only considering the transmitter noise and reference noise (which is 0), we have:



Note that our high and low noise margins are equal: $I_{Margin(high)} = I_{Margin(low)}$. Again, we want all of our noise sources to fit in our gross margin, I_{GM}. So, we have:

$$I_{GM} = I_1 \geq 0.15I_1 + 0.25(2I_1) - 600\mu A$$

$$\boxed{I_1 \geq 1.71mA}$$

Again, if we calculate the proportional noise compared to the worst-case signal on the aggressor ($1.15I_1$), we get:

$$\boxed{I_1 \geq 2.18mA}$$

3 Problem 7-6 (Dally and Poulton)

Rise-Time Control:

Consider the circuit of Figure 7-6. where Z_{RT} is a 5-nH inductor, $R_o = Z_o = 50\Omega$, and N = 4 outputs share a return lead. What is the signal-return cross talk coefficient if the output rise time is 100ps? What is the minimum rise time that gives a coefficient of 10% or less?

Using the equations derived in the problem set, we have:

$$K_{XRT} = \frac{Z_{RT}(N-1)}{Z_{RT}(N-1) + R_o + Z_o}$$

Solving for Z_{RT}, we get:

$$Z_{RT} = \frac{L_R}{t_r} = \frac{5nH}{100ps} = 50\Omega$$

Solving for K_{XRT}, we get:

$$K_{XRT} = \frac{50(3)}{50(3) + 100} = \boxed{0.6}$$

Now we solve for the minimum rise time that will give a coefficient of 10% or less. We have:

$$0.1 = \frac{\frac{5nH}{t_r}(3)}{\frac{5nH}{t_r}(3) + 100}$$

Solving for t_r we get:

$$\boxed{t_r = 1.35ns}$$

This result makes intuitive sense. As we increase the rise time, the impedance of the inductor decreases and thus reduces the amount of current injected into the victim lines.