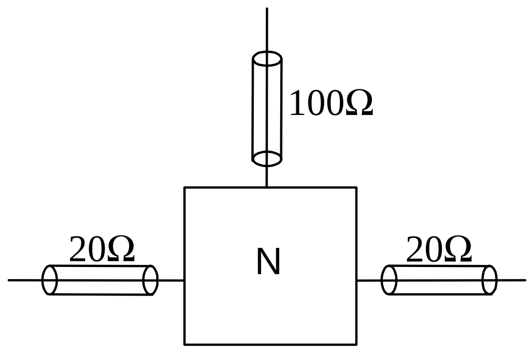
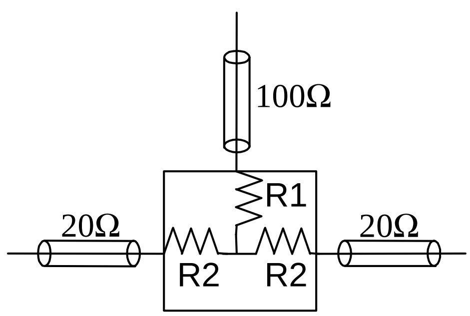
Homework 2 - Solutions

1 Problem 3-7 (Dally and Poulton)

Buses Without Stubs: One can build a bus (multidrop transmission line) in which there are no reflections off the stubs by placing matching networks at each drop across the line, as shown in Figure 3-57. (a)Design the resistive matching network, N, shown in the figure so that a signal transmitted to the network from any of its three terminals is propagated out the other two terminals (possibly attenuated) with no reflections. (b) With the impedance values shown in the figure, 20-Ω bus and 100-Ω stubs, how much energy is lost from the signal traveling down the bus at each stub? (c) How much energy would be lost if both the stubs and the bus were 50-Ω lines?



(a) This problem is very similar to Problem 3-6 that we solved in the previous homework. The simplest solution out of the plethora of possible ways is shown below.



Thus, we must solve for the following equivalences:

$$20\Omega = R_2 + [(R_2 + 20\Omega)//R_1 + 100\Omega)]$$

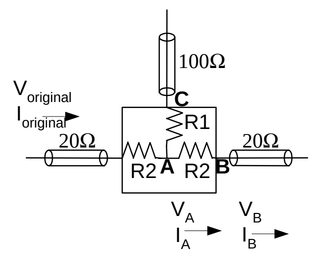
$$100\Omega = R_1 + [(R_2 + 20\Omega)//(R_2 + 20\Omega)]$$

Solving these equations simultaneously, we get:

$$\boxed{R_1 = 89.5\Omega}$$

$$\boxed{R_2 = 1.05\Omega}$$

(b) To solve for the energy lost at each of stubs, we solve for the current and voltage left after 1 stub.



We start by solving for the current and voltage at point A, shown in the figure above. The voltage at point A will be:

$$V_A = \frac{Z_1}{R_2 + Z_1} \times V_{original} = 0.947 \times V_{original}$$

$$, where Z_1 = (R_2 + 20\Omega)//(R_1 + 100\Omega) = 18.9\Omega$$

The current flowing from point A to point B will be:

$$I_A = \frac{R_1 + 100\Omega}{R_1 + 100\Omega + R_2 + 20\Omega} \times I_{original} = 0.9 \times I_{original}$$

The voltage and current at point B will be:

$$V_B = \frac{20\Omega}{20\Omega + R_2} \times V_{original} = 0.95 \times V_A = 0.89 V_{original}$$

$$I_B = I_A$$

Thus, the total power left to pass onto the next stub on the bus is:

$$P_{out} = I_B * V_B = 0.81 * P_{original}$$

So, the power lost at each stub as the signal travels down the line is:

$$\boxed{19\% \text{energy loss at each stub}}$$

We note that 5% of $P_{original}$ continues onto the 100Ω stub at point C, so the actual energy loss we have through our network is only $\boxed{14\%}$. However, the energy loss to the signal continuing down the rest of the bus is, as above stated 81% of $P_{original}$.

(c) Now, if the stubs and the bus were all 50 Ohm, we can go through the exact same procedure to find the resistor values, which turn out to be:

$$\boxed{R_1 = 16.7\Omega}$$

$$\boxed{R_2 = 16.7\Omega}$$

We note that since the network is now symmetric in three ways, we would expect that $R_1 = R_2$ as indeed our results show.

Again, using the same procedure as above, we get:

$$\boxed{75\%\text{energy loss at each stub}}$$

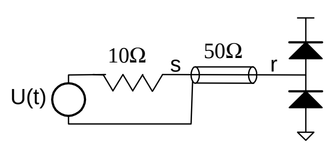
Or, in other words. Only 25% of the original energy, $P_{original}$, is available for the proceeding stubs. Again, as noted above, 25% of the original power also reaches point C in our circuit. So, only $\boxed{50\%}$ of our energy is consumed in our resistive network. However, only 25% of our original signal energy is available to continue on the bus to the rest of the stubs.

We can see the trade-off between the first and second networks. If there are many stubs on the bus, we want the maximum signal power available after passing each stub, as in the first setup. This is especially true if the transmitter stub is far away from the receiver stub. However, if there are few stubs, we want the maximum power to reach the stubs, as in the second setup.

Since a normal bus has a lot of stubs attached to it, the first setup of unequal impedances between stub and bus segment is generally a better idea.

2 Problem 3-10 (Dally and Poulton)

Nonlinear Termination: Consider the circuit of Figure 3-60. A voltage source with a 10-Ω output impedance drives a transmission line terminated into a pair of diodes (assume these are ideal diodes with no voltage drop) that restrict the range of the signal between 0 and 1 V. Sketch the waveforms that result at both ends of the line in response to a unit step on the voltage source.



The initial pulse traveling down the transmission line has the value of:

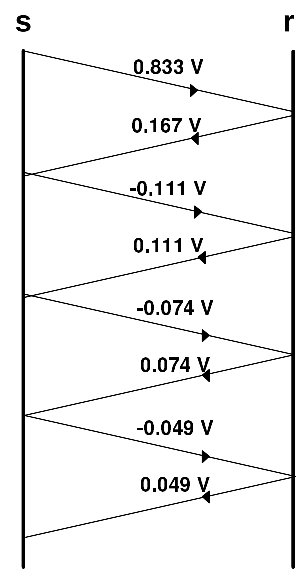
$$V_i(s,0) = 1V \times \frac{50}{50+10} = 0.833V$$

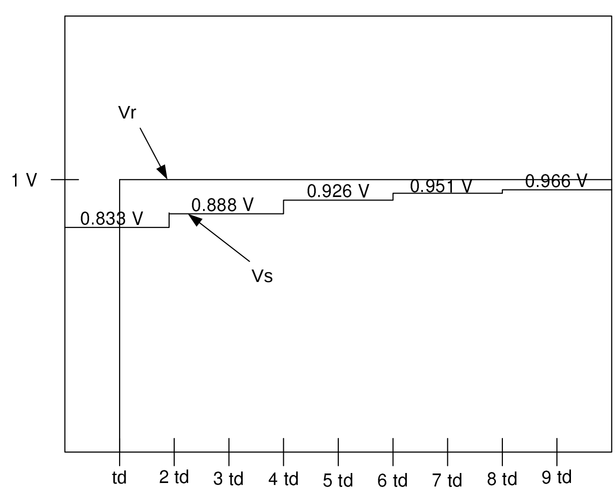
At the beginning of time, V(r) = 0V. As the 0.833 V wave reaches the diodes, **r** in the diagram, neither of the diodes is conducting since the voltage at **r** is neither greater than or equal to 1 V (when the top diode would begin conducting) or less than or equal to 0 V (when the bottom diode would begin conducting).

Thus, as this initial 0.833 V wave initially hits the end of the line it sees an open circuit since neither of the diodes is conducting.

Nominally for an open circuit, the forward wave would get reflected back with $K_r = 1$. But that would mean that V(r) = 1.66 V. We know, however, that once V(r)=1V, the top diode begins conducting, forcing V(r) = 1V. In other words, as the total voltage seen at r, the summation of the forward wave, $V_{forward}$ = 0.833 V, and the reverse wave $V_{reverse}$, becomes 1 V, the diode begins conducting and sinks the excess current. Thus, the wave reflected back to the source is 1 - 0.833V = 0.167V. As this reflected wave reaches the source, **s** in the figure above, it sees a negative reflection coefficient of -0.667, so the source then reflects a wave back toward the termination of $-0.667 \times 0.167 = -0.111$ V.

When this reflected wave reaches r, since the top diode is conducting, the reflection coefficient is that of a short: K_r = -1. So, this -0.111V wave gets reflected back as a positive wave of magnitude 0.111 V. This basically "rings" up the source voltage, while V(r) rings up to and gets clamped to 1 V with the incident wave. The bounce diagram and voltage plots are shown below.

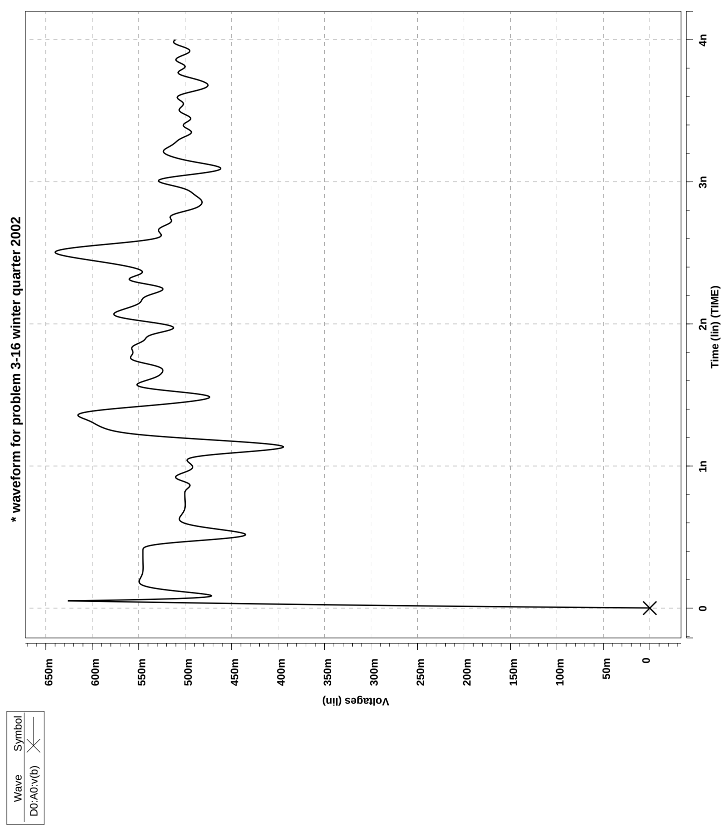




You can also consider the reflection process with currents as opposed to voltages ringing along the line. The inital pulse puts a current of $\frac{1V}{60\Omega} = 16.67mA$ into the line. At the far end, which is terminated by the diodes, 3.4 mA reflects back and 13.2 mA goes into the diode. When the 3.4 mA reaches the source, -2.2 mA reflects back. When this gets to the far end, it reduces the 13.2 mA to 11.0 mA. The diode is still conducting and a negative reflection of the current occurs – reducing the diode current to 8.8mA and so on.

3 Problem 3-16 (Dally and Poulton)

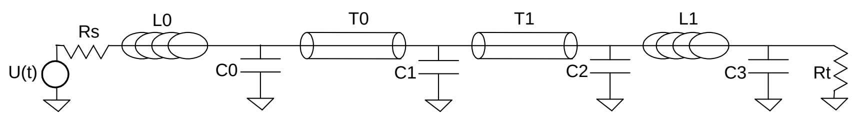
Extracting Parasitics: Develop a model circuit composed of ideal transmission lines, inductors, and capacitors that gives the same response as the given waveform. You will want to simulate your model circuit with HSPICE to verify correspondence.



There isn't a clean analytical way of generating the given waveform. We can start with a few equations that give rough estimates of the device values, but because of secondary effects, intermediate reflections, oscillations between inductors and capacitors, etc., eventually we have to resort to the brute force method of trying various parameters until we get a satisfying circuit.

The first step is to determine the arrangement of devices that will exhibit the same behavior as expressed by the waveform. Figure 3-47 of the textbook shows the reflection behavior of a large capacitor and a large inductor. Although we won't see this kind of clean exponential decay following an abrupt change in voltage, it can be observed that a positive bump is probably due to a series inductance, whereas a negative bump is probably due to a parallel capacitor.

We first approach the circuit with a broad brush. While there are many possibilities for modeling our circuit to obtain the desired waveform, we want to use the simplest circuit as possible. We could initially model our circuit as follows:



Using a source resistance of a TDR of 50 Ω and a rise time of 50ps (as estimated from the initial rise of the signal at t=0). We estimate the values of our discontinuities using the equations in the book, repeated here for convenience:

$$\tau_C = \frac{Z_o C}{2}$$

$$\tau_L = \frac{L}{2Z_o}$$

$$\frac{\Delta V}{V} = \frac{\tau}{t_r}[1 - exp(-\frac{t_r}{\tau})]$$

$$\tau = t_r \frac{\Delta V}{V}$$

Additionally, we calculate the impedance of the transmission line from the reflection coefficient, namely:

$$\frac{\Delta V}{V_{prev}} = \frac{Z_{Tnew} - Z_{prev}}{Z_{Tnew} + Z_{prev}}$$

Using these equations, we make our initial approximations as:

$$L_0 = 0.24nH$$

$$C_0 = 0.26pF$$

$$T_0 = 60\Omega, 140ps$$

$$C_1 = 0.62pF$$

$$T_1 = 50\Omega, 300ps$$

$$C_2 = 0.8pF$$

$$L_1 = 3.4nH$$

$$C_3 = 0.8pF$$

These values were found by approximation from the waveform given. To illustrate the approach, we document how we calculated these values for the first few components:

For L_0,

$$\tau = \frac{\Delta V}{V} \times t_r$$

While using the more detailed equation would give us more accuracy, we will use this method for the first approach. So, we get:

$$\tau = \frac{130mV}{550mV} \times 50ps = 11.8ps$$

$$\tau = \frac{L}{Z_{effective}}, where Z_{effective} = 50\Omega$$

Thus,

$$L_0 = \tau * 50\Omega = 0.24nH$$

Note, for this initial inductor, the initial rise is due partly to just the rise time of the signal and partly to the effect of the inductor. We estimate the effect of the inductor as above, but it is only an estimate, as are all of our calculations here. Similarly, we estimate the value, V, that the inductor finally reaches to be 550 mV, although you cannot see this value on the waveform, since the proceeding capacitor immediately starts pulling the voltage down as the current from the inductor reaches it.

Similarly for C_0 we have:

$$\tau = \frac{80mV}{550mV} \times 50ps = 7.27ps$$

We can either assume the inductor, L_0, has almost reached steady state to calculate the effective resistance that the capacitor, C_0 sees (i.e. C_0 sees the source resistance as well as the resistance of the following transmission line). Or we can assume the inductor L_0 is no where close to steady state and the capacitor cannot see the source resistance (i.e. it only sees the resistance of the proceeding transmission line). Either option will be an approximation.

We choose to assume that L_0 is near steady state and the capacitor can see the source resistance. Thus, we have:

$$C_0 = \frac{7.27ps}{Z_{effective}}$$

In order to find the value of the capacitor, we need to calculate the value of the characteristic impedance of the transmission line. First of all from the waveform, we see that the round-trip delay of the transmission line is about 275 ns. Specifically, the flat part of our waveform representing T_0 starts at approximately 175 ps and ends at approximately 450 ps, giving us a difference of (450-175ps) = 275ps. Thus, the delay of our transmission line, t_d is approximately $\frac{275ps}{2} = 137.5ps \approx 140ps$. We estimate the impedance of the transmission line by the equation above, where Z_{Tnew} is the impedance of the transmission line and Z_prev is the impedance of the source resistor, 50 Ω.:

$$\frac{\Delta V}{V} = \frac{Z_{Tnew} - Z_{T_prev}}{Z_{Tnew} + Z_{T_prev}} \times V_{prev}$$

$$\frac{50mV}{500mV} = \frac{Z_{Tnew} - 50}{Z_{Tnew} + 50}$$

Solving for the impedance of the transmission line, Z_{Tnew}, we get

$$Z_{Tnew} = 61\Omega \approx 60\Omega$$

Plugging this back into our capacitance equation above, we get:

$$C_0 = \frac{7.27ps}{Z_{effective}} = \frac{7.27ps}{50\Omega//60\Omega} = \frac{7.27ps}{27.27\Omega} = 0.26pF$$

Solving for C_1, we see a rise time of approximately 80ps - i.e. the falling edge from 440ps - 520ps. Thus, we see our rise time being degraded by the previous elements. Δ V is the drop from 545 mV to 440 mV, so Δ V = 105 mV. The final value that the capacitor reaches, V, is about 0.5 mV. So, we have:

$$\tau = \frac{105mV}{500mV} \times 80ps = 16.8ps$$

The effective impedance seen by the capacitor is the characteristic impedance of T_0 in parallel with the characteristic impedance of T_1. We calculate for the characteristic impedance of T_1 as:

$$\frac{-50mV}{550mV} = \frac{Z_{Tnew} - 60}{Z_{Tnew} + 60}$$

Solving for Z_{Tnew} we have,

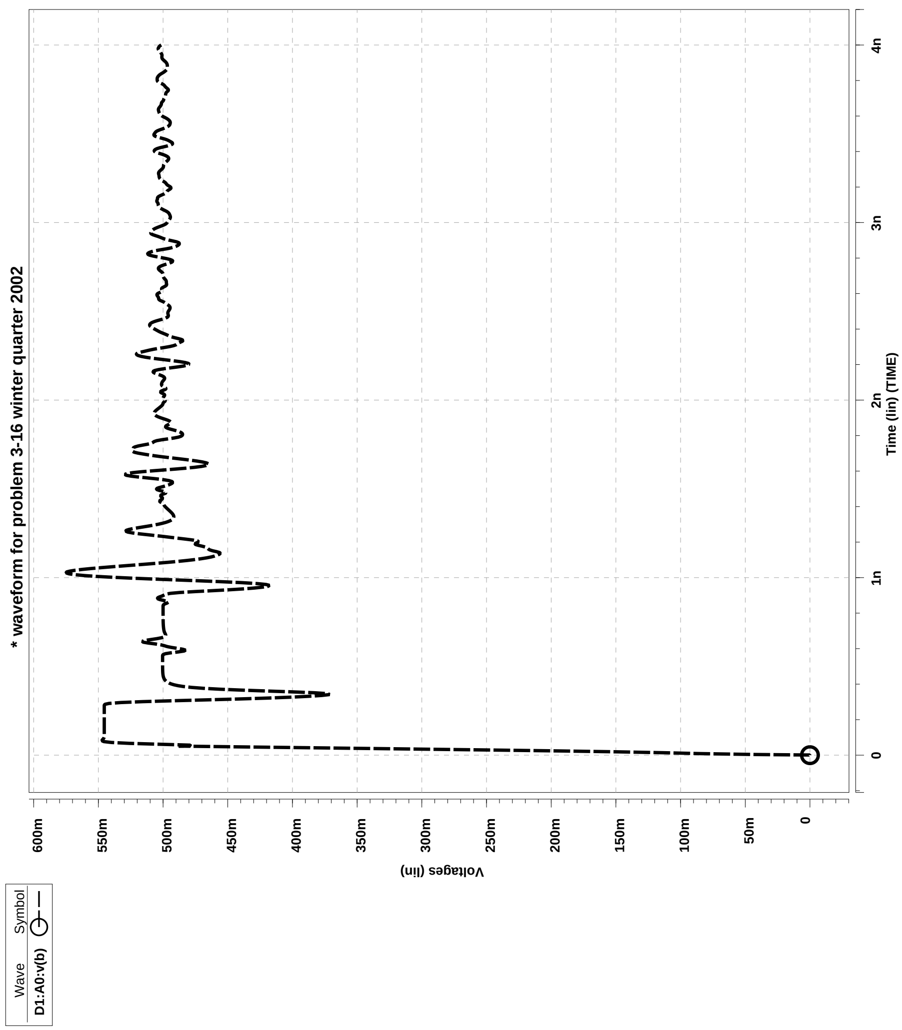
$$Z_{Tnewy} = Z_{T1} = 50\Omega$$

So, solving for C_1 we get:

$$C_1 = \frac{16.8ps}{60\Omega//50\Omega} = 0.62pF$$

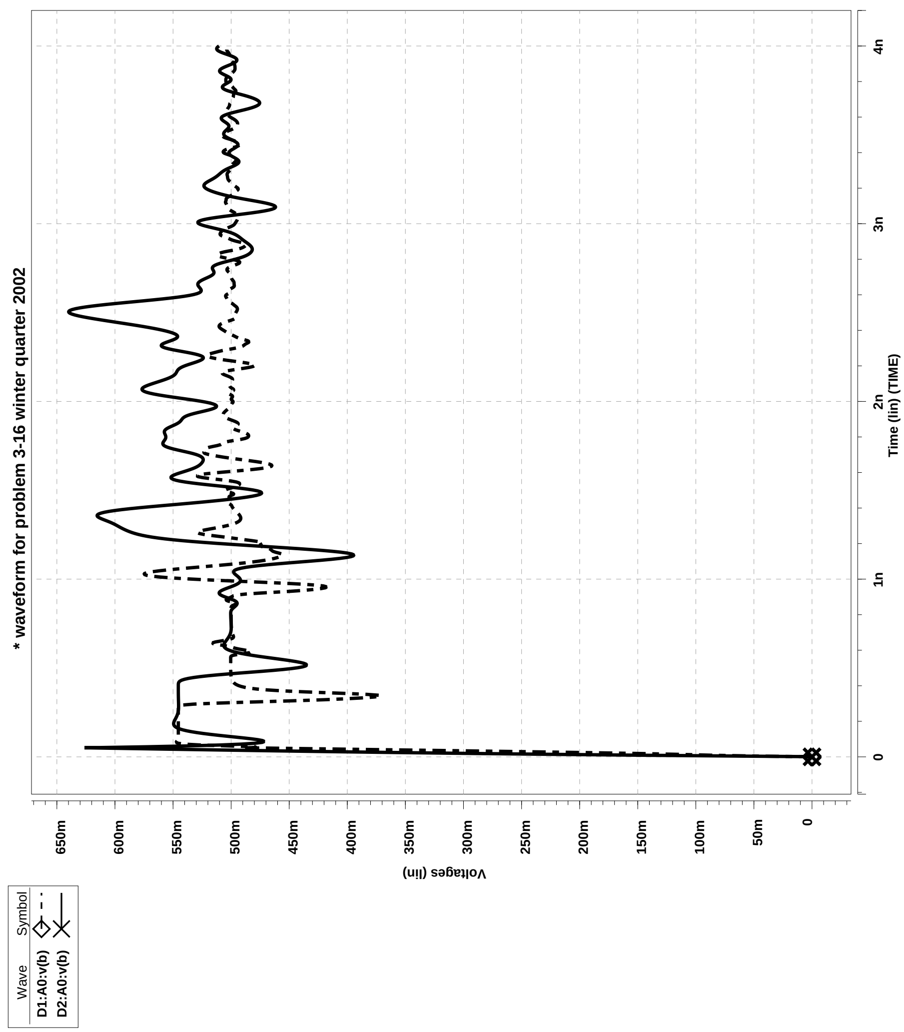
We continue with our estimates of component values in a similar fashion.

Next we spice our modeled circuit to see if it matches the given circuit/waveform. We get:



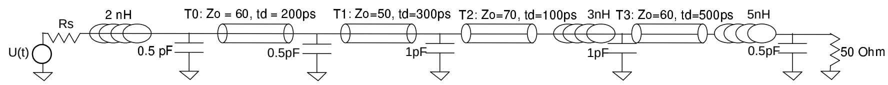
If we superimpose this waveform over the given waveform (see next page), we notice a few things: (1) Our initial inductor and capacitance values we calculated are so small that they're hardly affecting the signal - they're only managing to low-pass the incoming signal. Thus, we need to increase the values of L_0 and C_0.

(2)Also, we notice that our waveform is considerably more compact than the waveform given. Thus, we need to increase the delay of our circuit. This will mostly be done by increasing the delay of the transmission lines. The delay will also be effectively increased as we increase the inductance and capacitance in our circuit. (3) In the far end of the waveform, we notice that the average voltage of the original waveform raises above 500 mV, whereas our waveform simply stays around 500mV. We probably need to add a transmission line with a higher impedance than 50 Ω at this point.



With these initial observations, we begin adjusting our circuit by adding one component of our circuit at a time, from left to right, until our waveform looks like the one given.

Below is the original circuit:



We notice that the final 0.5pF capacitor is so small that you don't notice its effects in the given waveform. Therefore, in our model we don't need to model this last capacitor since if our TDR signal can't see it, neither willl our actual signals.