

# Homework 1 - Solutions

## 1 Problem 3-1 (Dally and Poulton)

**Example Transmission Lines:** Calculate the electrical properties ( $R_{DC}$ ,  $C$ ,  $L$ , and  $Z_o$ ) of the following common transmission media: (a) a twisted pair made of “wire-wrap” wire; (b) RG-58 coaxial cable (compare your results with the published values); (c) a wire-wrap wire glued to the surface of a PC board,  $\epsilon_r = 4.5$  and 6 mil dielectric thickness to first plane layer (for this one, you can either ignore that the line is inhomogeneous or find the appropriate empirical formulas in one of the references). For this exercise, go to the manufacturer’s data on common wire and cable to find the various physical and electrical parameters.

(a) twisted pair made of “wire-wrap” wire:

Using Coopertools wire-wrap wire with specifications (see <http://www.coopertools.com/>)

Part No.	AWG	Conductor	Insulator	Outer Diameter (mm)
990222	30	Copper	Teflon	0.5

$$R_{DC} = \frac{\rho l}{A}$$

$$L = \frac{\epsilon \mu}{C}$$

$$Z_o = \sqrt{\frac{L}{C}}$$

From Table 3-1 on page 82, we see that copper has a resistivity,  $\rho$ , of  $1.7 \times 10^{-8} \Omega - m$ . From Table 2-2 on page 50, we see that 30-Gauge wire has a diameter of 0.2548 mm ( $2.548 \times 10^{-4}$  m). Thus, the radius of the wire is  $0.5 * 2.548 \times 10^{-4} m = 1.274 \times 10^{-4} m$ . We calculate the DC resistance per unit length for one wire as:

$$R_{DC} = \frac{\rho}{A} = \frac{1.7 \times 10^{-8} \Omega - m}{\pi * (1.274 \times 10^{-4} m)^2} = 0.333 \frac{\Omega}{m}$$

The total DC Resistance of the twisted pair is twice the DC resistance of one wire. Thus, the DC resistance of the twisted pair is:

$$R_{DC} = 0.666 \frac{\Omega}{m}$$

Equation 3-5 on page 83 gives us the capacitance equation for 2 wires:

$$C_c = \frac{\pi \epsilon}{\ln\left(\frac{a}{r}\right)}$$

We find the relative permittivity,  $\epsilon_r$ , of the insulator, Teflon, in Table 3-2 on page 84:  $\epsilon_r = 2$ . Thus, we have:

$$C_c = \frac{\pi * 2 * 8.854 \times 10^{-12} \frac{F}{m}}{\ln\left(\frac{0.5 mm}{0.1274 mm}\right)} = 4.07 \times 10^{-11} \frac{F}{m} = \boxed{40.7 \frac{pF}{m}}$$

Equation 3-8 on page 84 gives us the equation:

$$C * L = \epsilon\mu$$

Thus, we have:

$$L = \frac{\epsilon\mu}{C} = \frac{2 * 8.854 \times 10^{-12} \frac{F}{m} * 4\pi * 10^{-7} \frac{H}{m}}{4.07 * 10^{-11} \frac{F}{m}} = 5.47 * 10^{-7} \frac{H}{m} = \boxed{0.547 \frac{\mu H}{m}}$$

Equation 3-31 on page 93 gives us the equation

$$Z_o = \sqrt{\frac{L}{C}}$$

Thus:

$$Z_o = \sqrt{\frac{5.47 * 10^{-7} \frac{H}{m}}{4.07 * 10^{-11} \frac{F}{m}}} = \boxed{116\Omega}$$

(b) RG-58 coaxial cable

Using Belden RG-58 coaxial cable with specifications (see: <http://www.belden.com/>)

Part No.	AWG	Conductor	Insulator	Outer Diameter (in)	Polyethylene $\epsilon$
9310	20	Bare Copper	Polyethylene	0.114	2.3

From Table 2-2 on page 50, 20-Gauge wire has a diameter of 0.8118 mm ( $8.118 * 10^{-4}$  m). Thus, the radius of the wire is  $0.5 * 8.118 * 10^{-4} \text{m} = 4.059 * 10^{-4}$  m. We calculate the DC resistance per unit length as:

$$R_{DC} = \frac{\rho}{A} = \frac{1.7 * 10^{-8} \Omega - m}{\pi * (4.059 * 10^{-4} \text{m})^2} = \boxed{0.0328 \frac{\Omega}{m}}$$

Note: we are ignoring the resistance of the return path, the outer conductor, since its resistance is negligible ( $4 * 10^{-3} \frac{\Omega}{m}$ ).

Equation 3-4 on page 83 gives us the capacitance equation for 2 wires:

$$C_b = \frac{2\pi\epsilon}{\ln(\frac{r_2}{r_1})}$$

$$r_2 = 0.5 * 0.114 \text{ in} = 0.057 \text{ in} = 1.45 \text{ mm}$$

Thus, we have:

$$C_b = \frac{2\pi * 2.3 * 8.854 \times 10^{-12} \frac{F}{m}}{\ln(\frac{1.45 \text{mm}}{0.406 \text{mm}})} = 1.01 * 10^{-10} \frac{F}{m} = \boxed{101 \frac{pF}{m}}$$

Equation 3-8 on page 84 gives us the equation:

$$C * L = \epsilon\mu$$

Thus, we have:

$$L = \frac{\varepsilon\mu}{C} = \frac{2.3 * 8.854x10^{-12} \frac{F}{m} * 4\pi * 10^{-7} \frac{H}{m}}{1.01 * 10^{-10} \frac{F}{m}} = 2.54 * 10^{-7} \frac{H}{m} = \boxed{0.254 \frac{\mu H}{m}}$$

Equation 3-31 on page 93 gives us the equation

$$Z_o = \sqrt{\frac{L}{C}}$$

Thus:

$$Z_o = \sqrt{\frac{2.54 * 10^{-7} \frac{H}{m}}{1.01 * 10^{-10} \frac{F}{m}}} = \boxed{50.3\Omega}$$

Comparing our values with the values published by Belden, we have

Source	$R_{DC}$	Capacitance	Inductance	Impedance
Belden	$9.4 \frac{\Omega}{1000ft} = 0.0308 \frac{\Omega}{m}$	$30.8 \frac{pF}{ft} = 101 \frac{pF}{m}$	$0.09 \frac{\mu H}{ft} = 0.295 \frac{\mu H}{m}$	50 $\Omega$
Calculated	$0.0328 \frac{\Omega}{m}$	$101 \frac{pF}{m}$	$0.254 \frac{\mu H}{m}$	50.3 $\Omega$

Therefore, our calculated values are different from the published values by, 6.5%, 0%, -13.9%, and 0.6% respectively.

(c) a wire-wrap wire glued to the surface of a PC board,  $\varepsilon_r = 4.5$  and 6 mil dielectric thickness to first plane layer. (note: 1000 mil = 1 inch)

We will ignore that the line is inhomogeneous.

From (a) we know that the DC resistance of the wire is:

$$R_{DC} = \boxed{0.333 \frac{\Omega}{m}}$$

Note: We are ignoring the resistance of the return path, the ground plane, since its resistance is negligible. Equation 3-6 on page 83 gives us the capacitance equation for a single wire over a ground plane:

$$C_d = \frac{2\pi\varepsilon}{\ln(\frac{2s}{r})}$$

$r = 0.1274$  mm  $s = 0.25$ mm (half the diameter of the entire wire) + 0.006 in. = (0.25 + 0.1524)mm = 0.402 mm

The electric field lines go through Teflon, the dielectric of the board, and air. We ignore the inhomogeneity of the dielectric, and take the relative dielectric constant to be that of Teflon.

Thus, ignoring the inhomogeneity of the insulator, we have:

$$C_d = \frac{2\pi * 2 * 8.854x10^{-12} \frac{F}{m}}{\ln(\frac{2*0.402mm}{0.1274mm})} = 6.04 * 10^{-11} \frac{F}{m} = \boxed{60.4 \frac{pF}{m}}$$

Solving for the inductance, we get:

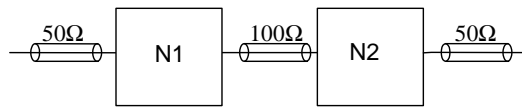
$$L = \frac{\varepsilon\mu}{C} = \frac{2 * 8.854x10^{-12} \frac{F}{m} * 4\pi * 10^{-7} \frac{H}{m}}{6.04 * 10^{-11} \frac{F}{m}} = 3.68 * 10^{-7} \frac{H}{m} = \boxed{0.368 \frac{\mu H}{m}}$$

Solving for impedance, we get:

$$Z_o = \sqrt{\frac{3.68 \times 10^{-7} \frac{H}{m} F}{6.04 \times 10^{-11} \frac{m}{m}}} = \boxed{78.1\Omega}$$

## 2 Problem 3-6 (Dally and Poulton)

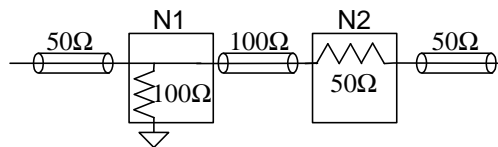
**Resistive Matching Networks:** One can propagate a signal between transmission lines of differing impedance without reflections by inserting a matching network between the two lines. Consider the situation in Figure 3-56 where a signal is transmitted first over a 50- $\Omega$  line, then a 100- $\Omega$  line, then back to a 50- $\Omega$  line. (a) Using only resistors, design the networks, N1 and N2, so that there are no reflections from a wave traveling from left to right. (b) Now modify your design so that it works for a wave traveling in either direction. (c) How much signal level is lost passing through the two networks? How much signal energy?



(a) Looking into N1, we want to match the impedance of the 50- $\Omega$  transmission line. Thus we want the N1 network in parallel with the 100- $\Omega$  transmission line that follows it to equal 50  $\Omega$ s.

Thus, we put in a 100- $\Omega$  resistor connected to ground for N1. The 100- $\Omega$  resistor in parallel with the 100- $\Omega$  transmission then look like 50  $\Omega$ s to the preceding transmission line.

Similarly, we want N2 and the preceding 50- $\Omega$  transmission line to look like 100  $\Omega$ s to match the preceding 100- $\Omega$  transmission line. Thus, for N2 we put in a 50- $\Omega$  series resistor.



(b)

Now, we solve for no reflections in the forward and reverse directions:

For the first network, we need N1 and the 100- $\Omega$  transmission line to look like 50  $\Omega$ s (as before). But now we also require that N1 and the 50- $\Omega$  transmission line (looking in the reverse direction) look like 100  $\Omega$ s.

If we put a resistor connected to ground,  $R_1$ , followed by a series resistance,  $R_2$ , we can solve for their values given the above constraints, namely:

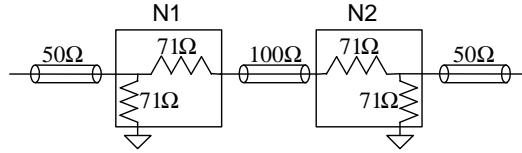
$$(R_1 + 100\Omega) // R_2 = \frac{1}{\frac{1}{R_1 + 100\Omega} + \frac{1}{R_2}} = 50\Omega$$

$$R_1 + (R_2 // 50\Omega) = 100\Omega; \text{ thus, } \frac{(R_1 + 100\Omega)R_2}{R_1 + R_2 + 100\Omega} = 50\Omega$$

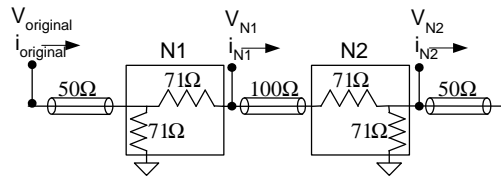
Solving these two equations simultaneously we get:

$$R_1 = R_2 = \sqrt{5000} = 70.7\Omega$$

And since  $N_1$  and  $N_2$  are symmetric, we have the symmetric circuit for  $N_2$ .



(c)



We will solve for the signal levels in the forward direction. For the current, the current continuing onto the 100  $\Omega$  transmission line is:

$$i_{N1} = \frac{70.7\Omega}{70.7\Omega + 70.7\Omega + 100\Omega} * i_{original} = 0.292 * i_{original}$$

The current then continuing onto the final 50- $\Omega$  transmission line is:

$$i_{N2} = \frac{70.7\Omega}{70.7\Omega + 50\Omega} * i_{N1} = 0.586 * i_{N1} = 0.586 * (0.292 * i_{original}) = 0.172i_{original}$$

For the voltage, we have  $V_{N1}$ , the voltage at the beginning of the 100- $\Omega$  transmission line as:

$$V_{N1} = \frac{100\Omega}{100\Omega + 70.7\Omega} * V_{original} = 0.586 * V_{original}$$

The voltage at the beginning of the final 50- $\Omega$  transmission line is then:

$$V_{N2} = \frac{(50\Omega//70.7\Omega)}{70.7\Omega + (50\Omega//70.7\Omega)} * V_{N1} = \frac{29.3\Omega}{70.7\Omega + 29.3\Omega} * V_{N1} = 0.292 * V_{N1} = 0.172 * V_{original}$$

Thus, only **17.2%** of our original signal is left after passing through the two networks. Thus, there is an **82.8%** signal loss. We note that, since the network is symmetric, the voltage and current loss is the same. It follows that the signal energy left after passing through the two networks is:

$$P = i * v = (0.172i_{original}) * (0.172V_{original}) = 0.03 * i_{original}V_{original} = 0.03P_{original}$$

Thus, we lose **97.1%** of our signal energy passing through the two networks.

Since the network is symmetric, the signal level and signal losses will be the same in either direction.

$$\boxed{SignalLevelLoss = 82.8\% \quad EnergyLoss = 97.1\%}$$