EE273 Digital Systems Engineering
Midterm Exam

February 9th, 2000

(Total time = 120 minutes, Total Points = 100)

Name: (please print)______________________________

In recognition of and in the spirit of the Stanford University Honor Code, I certify that I will neither give nor receive unpermitted aid on this exam.

Signature:_______________________________________

This examination is open notes open book. You may not, however collaborate in any manner on this exam. You have two hours to complete the exam. Please do all of your work on the exam itself. Attach any additional pages as necessary.

Before starting, please check to make sure that you have all 10 pages.

1		**30**
2		**20**
3		**20**
4		**15**
5		**15**
Total		**100**

Problem 1: Short Answer (30 Points: 10 questions, 3 points each)

A. Suppose you have a *parallel plate* transmission line (a pair of flat conductors that are wide enough that you can ignore the fringing fields – their capacitance is well approximated by the parallel plate component) with a characteristic impedance of 100Ω and a velocity of 1.5×10^8m/s. If you double the spacing between the two lines. What happens to the impedance of the line?

*In a homogenous medium(dielectric completely surrounds the conductors), we know that C*L=em.. In this situation, when we increase the spacing between the lines, the capacitance decreases by 2.* $C = \dfrac{e^* \, e0 * W*L}{s}$ ← *This has increased by 2, so C decreases by 2*

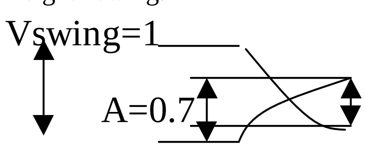
In order to keep em constant, L must increase by 2.

We know $Z = \sqrt{\dfrac{L}{C}}$ *So Z goes up by a factor of 2.*

B. For the transmission line of (A), what happens to the impedance when the width of the lines is doubled?

As above, *C*L=em.. if we double the width of the lines, C goes up 2, as described in (A). Therefore L, goes down by 2. From* $Z = \sqrt{\dfrac{L}{C}}$, *we will find an impedance that decreases by a factor of 2.*

C. Consider a 2Gb/s signaling system. At 1GHz the transmission line used by the system has an attenuation of A=0.7. There is essentially no attenuation, A=1, at DC. Without equalization, what will the vertical eye opening be for a worst case bit pattern. Express your answer as a fraction of signal swing.



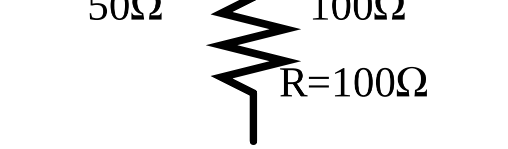
2A-1=0.4*Vswing

D. In an on-chip RC transmission line using optimal repeater spacing, if the capacitance per unit length of the wire is halved (and the repeater spacing adjusted to still be optimal), by what amount and in which direction does the delay per unit length change?

The velocity goes as $v = 1.3 * (t_b * RC)^{-0.5}$ ➔ *td=*$\dfrac{distance}{v} = \dfrac{d*(tb*RC)^{\wedge}0.5}{1.3}$

If C now decreases by a factor of 2, then the delay td will be reduced by a factor of sqrt(2)=0.707, as compared to the original case.

E. You wish to connect a 50Ω transmission line to a 100Ω transmission line. Sketch a resistor network that you could place between the two lines that will allow waves to propagate from one line to the other in just one direction (from the 50Ω line to the 100Ω line) without reflections.



F. You have designed a signaling system with a BER of 10^{-12}. If the Gaussian noise sources in this system are halved while everything else is held constant what will the new BER be?

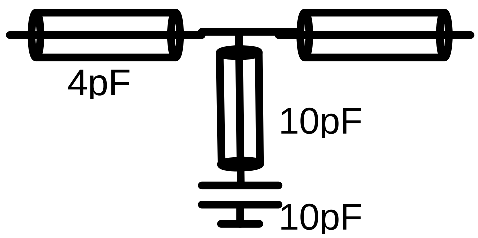
$BER = \exp\left(-\frac{VSNR^2}{2}\right) = 10^{-12}$. ➔ Solving for $VSNR = \frac{Vm}{Vr} = 7.43$.

*If we reduce the noise Vr by a factor of 2, then VSNR=7.43*2=14.867.*

➔ $BER = \exp\left(-\frac{14.687^2}{2}\right) = 1e-48$.

G. You are designing a backplane bus. The unloaded bus and stub traces have a capacitance of 100pF/m and an inductance of 300nH/m (Z_0 = 55Ω, v=1.8 x 10^8 m/s). The spacing between stubs is 4cm and the stubs are 10cm long and terminated in a 10pF load. What is the effective impedance of the loaded bus traces?

From lecture, we can treat the stubs as being lumped elements. That is, we lump the capacitance of the stubs into the characteristic impedance of the backplane bus.



Our old Capacitance of our tranmission line was Cold=4pF. Now, we have a stub, which we now lump into the characteristic impedance of the line. Since the stub is 10cm long, it provides an additional 10pF of capacitance. Along with the lumped 10pF at the far end, we now have a total of Cnew=24pF.

➔ $Z0 = \mathrm{sqrt}(L/C) = \mathrm{sqrt}\left[\frac{300nH*0.04}{24pF}\right] = 22W$

H. What is the fastest rise time that you can safely transmit over the backplane bus of (G)?

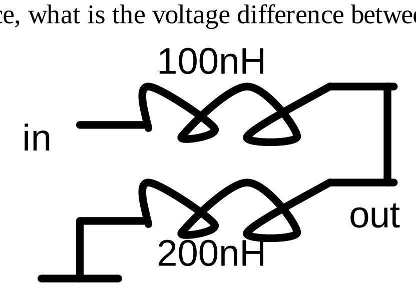
$Tstub = \frac{0.1m}{1.8e8m/s} = 0.55ns$

$Vbus = \frac{1}{Sqrt(L*C)} = \frac{1}{sqrt(300nH/m*600pF/m)} = 7.45e7m/s$

600pF/m comes from C/m = Ctot/length = 24pF/4cm = 600pF/m

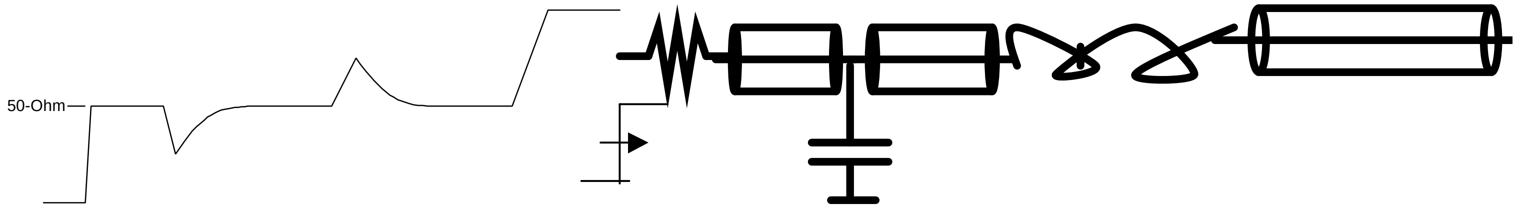
*$Tbus = \frac{0.04m}{7.45e7m/s} = 0.54ns$, ➔ trise(min)=5*max(Tstub, Tbus)=2.75ns*

I. You transmit a 1V step into a 10m cable that has a signal conductor with an inductance of 200nH/m and a return conductor with an inductance of 100nH/m. Just after the transition at the source, what is the voltage difference between the two ends of the return conductor?



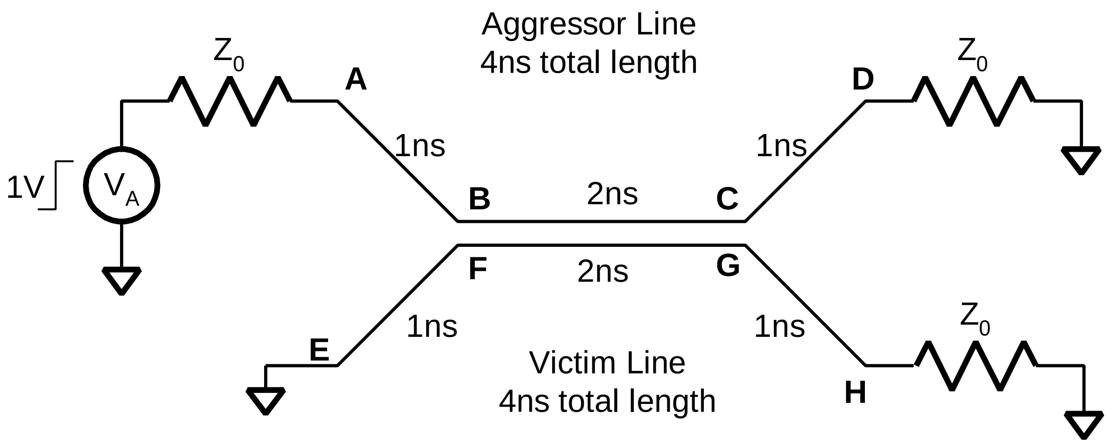
Effectively, the center conductor looks like 100nH, while the return conductor is a lumped 200nH. The voltage potential at V(in)=1V, and at V(out), Is simply a voltage divider➔ V(out)=2/3V. So the voltage difference is 1/3V.

J. A TDR of a transmission line shows the following waveform. Sketch qualitatively (no component values) a possible model for the line.



Problem 2: Transmission Lines (20 Points)

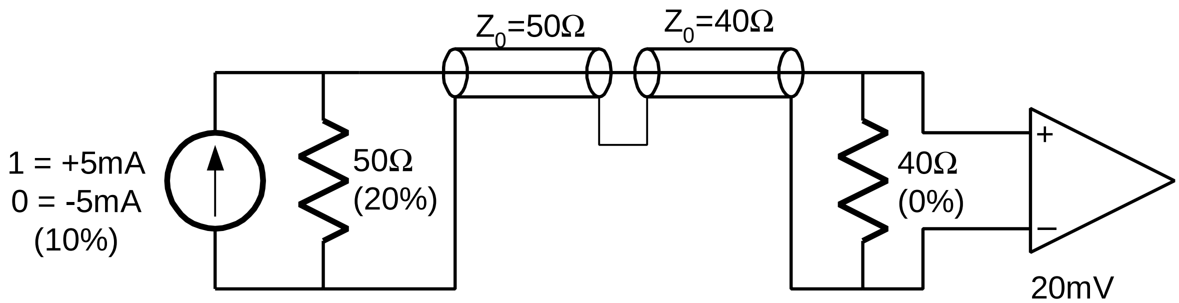
Consider the pair of coupled transmission lines shown below. The coupled section of each line has a near-end crosstalk coefficient k_{rx} of 0.1 and a far-end crosstalk coefficient, k_{fx} of 0. The aggressor line is driven directly by a 1V step source with a rise time of 100ps. The far end of the aggressor is terminated in a matched impedance. The victim line is shorted to the ground plane at the source end and terminated with a matched impedance at the far end. Note that the 'ground' symbol here denotes a local connection to the shared ground plane, not a single-point ground.



Using this information, sketch and dimension the voltage waveform at the far-end of the victim line (point H). You may ignore any effects that lead to waves with less than 10mV amplitude.

Problem 3: Signaling and Noise Analysis (20 points total)

Consider the bipolar current-mode signaling system shown below. At nominal levels, a logic "1" is represented with 5mA of current drive and a logic 0 is represented with -5mA of drive. The actual transmitter levels are within 10% of these nominal levels. The line is terminated at both ends into matched impedances with 20% tolerance at the source end and a perfect match[1] at the receiver. The line itself has a 20% impedance discontinuity (from 50Ω to 40Ω – note the length of these segments does not matter). The receiver has a combined sensitivity and offset voltage of 20mV. In addition, there is a 20mV Gaussian noise source (not shown) adding noise to the line.



A. (10 points) List all of the relevant noise sources for this system and compute the net noise margin. (Hint: you may ignore all forms of crosstalk, you may ignore all reflections after the second bounce, and may ignore the effect of the mismatched source termination on the magnitude of the incident wave).

The spreadsheet below lists the relevant calculations.

First, note that the impedance seen by the transmitter is 25-Ohms, the parallel combination of the line and the source termination. Thus, with a delta-I of 10mA, our signal swing is 250mV.

The first noise source is transmitter offset. This is 10% of 5mA or 0.5mA. Across the 25-Ohm source impedance this is 12.5mV (rounded in the table). Note that we ignore the effect of the mismatched source impedance on the magnitude of the incident wave.

Next we consider the effect of the 50-Ohm to 40-Ohm line discontinuity. Applying the Telegrapher's equation shows us that the magnitude of the incident wave is reduced by 1/9 at this point. We also note that the attenuated waveform is centered on GND. That is, even with a long string of 0s, our signal after this point will not go below –(8/9)(125) = –111mV and after a long string of ones, our signal will not go above (8/9)(125). Thus, our reduction in margin is (1/9)(125mV) or 14mV (not twice this amount which you would calculate if you thought of this as frequency-dependent attenuation, which is not centered).

The next component is the reciever offset+sensitivity which is 20mV.

The final component of bounded noise (which is optional) is the intersymbol interference due to the bounce off the 50-Ohm to 40-Ohm discontinuity and then the bounce off the source mismatch. Finally, this interference is attenuated by traveling through the 50-Ohm to 40-Ohm discontinuity. Consider all

[1] This is not realistic but it simplifies the problem.

three effects, the magnitude is 250mV(1/9)(1/9)(8/9) = 2.7mV.

The final relevant noise source is the 20mV RMS Gaussian source. However this is not a bounded source and so does ***NOT*** *go into our calculation of net margin.*

Tthe Gross margin is $\Delta V/2 = 125mV$*. From this we subtract the 49mV of bounded noise to get a net margin of 76mV.*

ΔV	250 mV	signal swing at transmitter
Vtx	13 mV	deviation from nominal transmitter value
Va	14 mV	absorbed at impedance discontinuity
Vrx	20 mV	receiver offset
Visi	3 mV	bounce from discontinuity and source
Vbn	49 mV	bounded noise
Vgm	125 mV	gross margin
Vnm	76 mV	net margin
Vg	20 mV	gaussian noise (RMS)
VSNR	3.78	voltage signal to noise ratio
BER	8.01E-4	

B. (5 points) Compute the BER for this signaling system.

As shown in the table above, the BER is calculated using the exponential approximation of the error function $BER = exp(-VSNR^2/2)$.

C. (5 points) Suppose 50mV of hysteresis is added to the receiver. That is its decision point rather than being at 0V is +50mV when the output is low and –50mV when the output is high. What are the values of the net noise margin and the BER with this hysteresis?

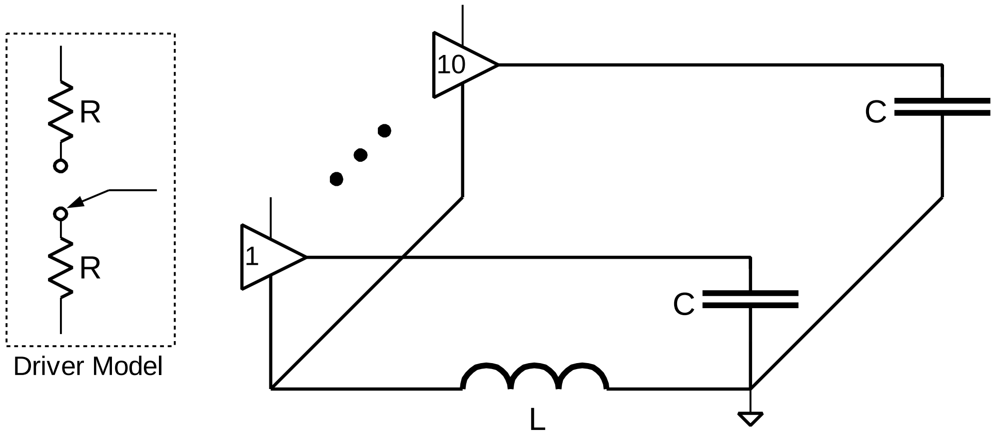
Adding hysteresis increases the noise margin for half the bits (the half that are the same as the previous bit) and decreases it for the other half (the half that are different). Unfortunately the decrease dominates. For these bits we have:

Vnmh	26 mV	net margin with hysteresis
VSNRh	1.29	VSNR with hysteresis
BERh	4.33E-1	BER with hysteresis

Strictly speaking, the BER his half this amount, 2.17×10^{-1} *since only half the bits are subject to this high error rate and the other half actually have a much better error rate. Still, the point is that adding hysteresis makes the signaling system worse, not better.*

Problem 4: Signaling over Lumped Loads (15 Points Total)

Consider the system shown below for signaling over a medium that is modeled as a lumped capacitor. Ten drivers share a single ground return pin with inductance L = 5nH. Each is loaded with a C = 10pF capacitor.

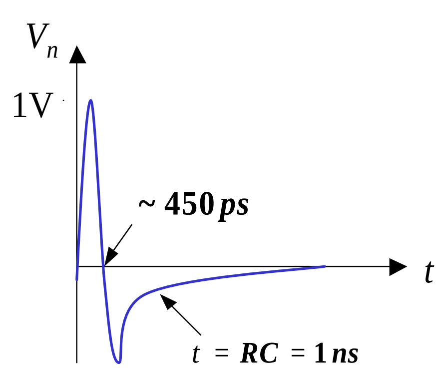


A. Suppose we use a driver that is accurately modeled as a switch and two resistors with value R = 100Ω as shown in the box at the left of the figure. Consider the case where nine of the drivers are initially high with outputs at 1V while the tenth driver has a low 0V output. What happens to the voltage on the output of the tenth output when the nine high outputs simultaneously switch from the high resistor to the low resistor? Sketch the resulting waveform. (Hint: the current waveform from this driver in the configuration shown above is plotted on page 9).

Note that the current waveform given in the next page is just an example for only one of the nine switching drivers is connected to the unswitched line, so you need to figure out what the actual current waveform will look like first.

When the 9 drivers switch from high to zero, the circuit behaves like an R-L network (the capacitors won't affect the circuit very much because of high frequency components in the fast transition). The equivalent resistor seen from the inductor is R = 100/9 = 11 W, i.e. the inductor sees 9 resistors in parallel. Thus the time that the whole network approximately looks like a R-L line is t_{LR} = L/R = 5 nH / 11 W = 450 ps, and the peak current will reach 1 V / 11 W = 90 mA.

*From the current waveform, we can calculate the voltage across the inductor by taking its derivative, or the voltage drop across the inductor is L*di/dt. Thus, for t<450ps, the voltage across the inductor goes instantly to its maximum value of 5nH*90mA/450ps = 1V, and when the current reaches its maximum, the voltage drops to 0.*



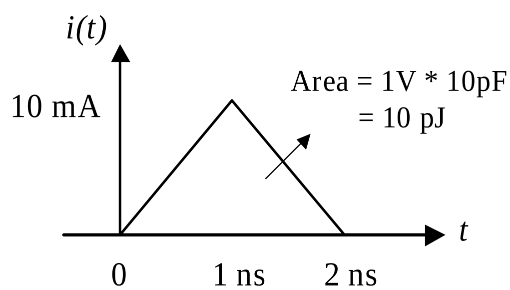
*After 450 ps, the circuit will behave like RC network, so the current will decay with a RC constant of t_{RC} = RC = 100 W * 10 pF = 1ns. Here we use R=100 W because each C will see individual R. If we take the derivative of this current, it gives again an exponential waveform with the same time constant but with negative amplitude, resulting the voltage waveform as in the figure above.*

Also noticing that the current doesn't continue resonating, the voltage will also not resonate. Essentially what have done is treated this 2^nd order system as a first order system. First, at the initial moment that we see the input voltage spike, we assumed the capacitor was a short and you only see the inductor. Therefore, we assumed a first order system with just the inductor and the resistor. Later, after time, we assumed that the inductor was a short and that the system was only the resistor and the capacitor. We were able to assume this information, if we calculate the Q of our system. From equation 7-36), $\boldsymbol{Q = \frac{1}{pR}\sqrt{\frac{L}{C}} = 0.2}$ *, where L=5nH, C=10pF*9=90pF, and R=100/9=11W One definition of Q is that it is the number of oscillations until the value settles. Since Q=0.2 is much less than 1(less than critically damped 0.7), then we know there should not be any oscillations in the voltage domain.(or only 0.2 Oscillations) So our assumptions above are in fact, correct.*

B. Suppose instead of the resistor and switch driver that you can substitute a driver that produces an arbitrary current waveform. What is the optimum driver current waveform that minimizes noise on the unswitched output while still switching within 2ns? Sketch the waveform.

The optimum current waveform will be in a triangular shape which gives a constant noise voltage across the inductor over the whole discharging time. Since this current should discharge the capacitor during 2 ns, we can get the peak value of the waveform as:

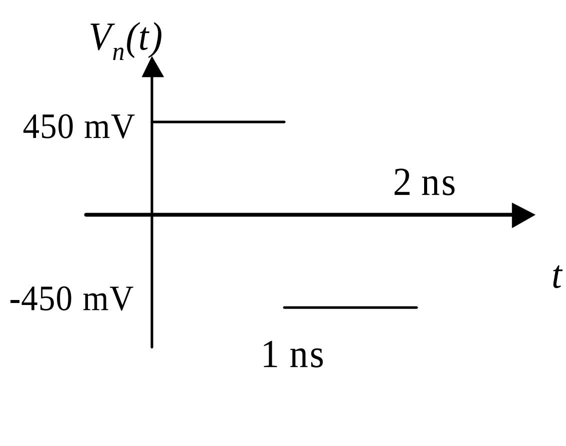
$$\boldsymbol{Area\ under\ i(t) = \frac{1}{2} \times 2ns \times I_{peak} = total\ charge\ on\ C = 1V \times 10\,pF = 10\,pJ}$$

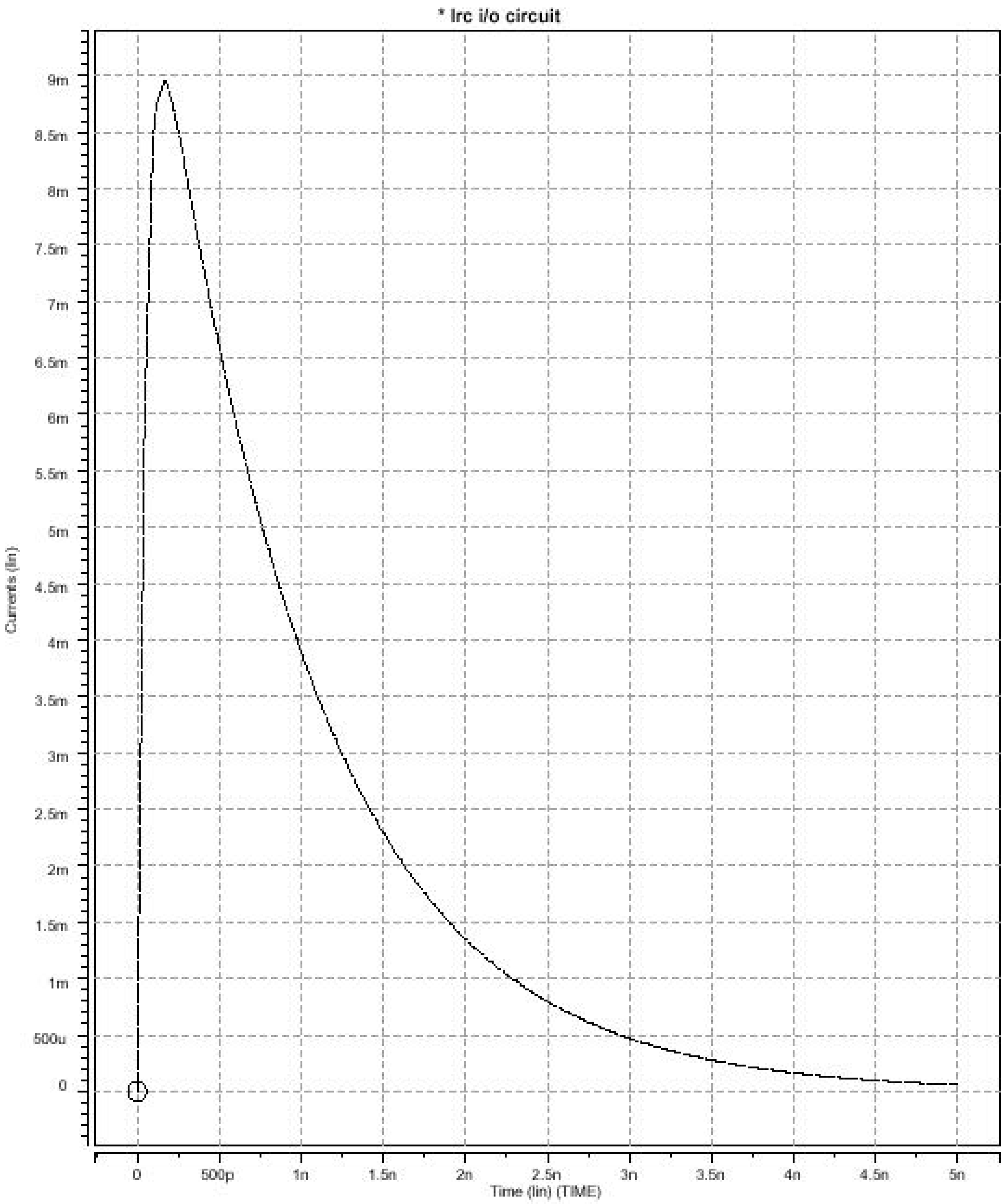


C. What is the noise on the unswitched output when nine drivers simultaneously switch low using the current profile from (B)?

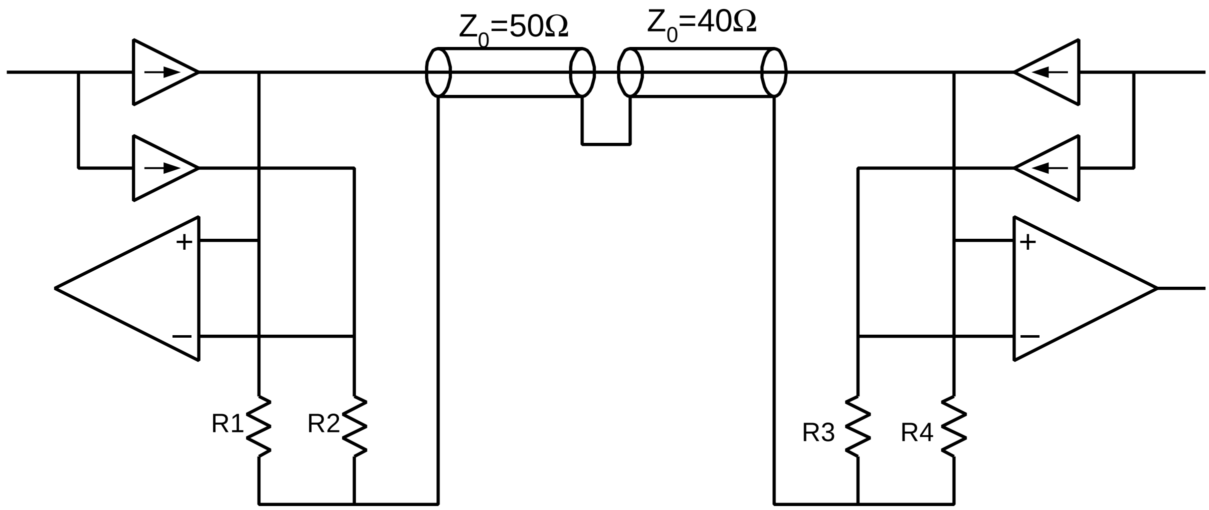
Nine times the current in part (B) will flow through the inductor to produce the supply noise at the unswitched output line as :

$$\boldsymbol{\left|V_n\right| = 9 \times L \times \left|\frac{di}{dt}\right| = 9 \times 5\,nH \times \frac{10\,mA}{1\,ns} = 450\,mV}$$




* lrc i/o circuit
9m
8.5m
8m
7.5m
7m
6.5m
6m
5.5m
5m
4.5m
4m
3.5m
3m
2.5m
2m
1.5m
1m
500u
0
Currents (lin)
0
500p
1n
1.5n
2n
2.5n
3n
3.5n
4n
4.5n
5n
Time (lin) (TIME)

Problem 5: Advanced Signaling (15 Points)



The figure above shows a simultaneous bidirectional signaling system that shares all parameters (transmit levels +/- 5mA (10%), receiver offset+sensitivity = 20mV, and resistor tolerances 20%) with the signaling system of Problem 3. For simplicity, the replica current drivers here are full-scale. That is each of the four drivers in the figure above drives a nominal value of +5mA for a 1 and –5mA for a 0. Also for simplicity, assume that resistors R2 and R4 are exact while R1 and R3 have a tolerance of 20%.

A. What are the proper nominal values for resistors R1 through R4?

   R1 = 50-Ohms and R4=40 Ohms to match their respective line segments. R2=25-Ohms to match the parallel combination of the line and R1. Similarly R3=20-Ohms to match the parallel combination of the line and R4.

B. Compared to the system of problem 3, what new noise sources and modes must this system contend with? List the sources and give the magnitude of each source. Consider just noise sources affecting the signal traveling from left to right.

(Hint: as in Problem 3, you may ignore all forms of crosstalk with other lines, you may ignore all reflections after the second bounce, and you may ignore the effect of the mismatched termination on the magnitude of the incident wave).

As illustrated in the table below there are three new noise source all of which represent different forms of reverse channel crosstalk. They are (1) reverse channel crosstalk due to the mismatch of R3 with R4||Z_0, (2) reverse channel crosstalk due to the reflection off the 40-Ohm to 50-Ohm discontinuity, and (3) reverse channel crosstalk due to the reflection off of R1. I assumed that two transmitters on the same chip are perfectly matched. If you considered the transmitters to vary 10% even on the same chip, you got a fourth source of reverse channel crosstalk which is OK but not needed for full credit.

To compute the magnitude of the first source, we note that the transmitter current is nominally +/- 5mA (if you used 5.5mA that's fine) and R3is 20-Ohms (20%), so the 100mV across R3 can be +/- 20mV

To compute the magnitude of the reverse channel crosstalk due to reflections, we first compute the magnitude of the reverse channel wave. ΔVR*, the signal swing in the reverse (right-to-left) direction is (10mA)(20-Ohms) = 200mV. With a reflection coefficient of 1/9 at the interface, the magnitude of the second component is (200mV)(1/9)=22mV.*

To calculate the third component, we follow the wave through the system. It is amplified by (10/9) at the 40-Ohm to 50-Ohm interface, then (1/9) of this value reflects from R1, and finally it is attenuated by (8/9) passing back through the 50 to 40 interface. The net result is 200mV(10/9)(1/9)(8/9) = 22mV.

ΔVL	250 mV	swing at left transmitter
ΔVR	200 mV	swing at right transmitter
Vrcx1	20 mV	reverse channel crosstalk due to mismatch of R3
Vrcx2	22 mV	reverse channel crosstalk due to bounce from line discontinuity
Vrcx3	22 mV	reverse channel crosstalk due to bounce from R1
Vrcx	64 mV	total reverse channel crosstalk
Vgm	125 mV	gross margin L-R channel
Vn	114 mV	total bounded noise
Vnm	11 mV	net margin
Vg	20 mV	gaussian noise (RMS)
VSNR	0.57	voltage signal to noise ratio
BER	8.51E-1	

C. What is the net noise margin of this system for the signal traveling from left to right?

As shown in the table above, the new noise sources total64mV. Starting with our old margin of 76mV, this leaves 11mV (after rounding). Because this is smaller than the Gaussian noise, the channel is very unreliable.